

# PAPERS ON THE SCIENCE OF ADMINISTRATION

BY

LUTHER GULICK, L. URWICK, JAMES D. MOONEY  
HENRI FAYOL, HENRY S. DENNISON, L. J. HENDERSON  
T. N. WHITEHEAD, ELTON MAYO, MARY B. FOLLETT  
JOHN LEE, V. A. GRAICUNAS

*Edited by*  
LUTHER GULICK and L. URWICK

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## X

### RELATIONSHIP IN ORGANIZATION

It has long been known empirically to students of organization that one of the surest sources of delay and confusion is to allow any superior to be directly responsible for the control of too many subordinates. Armies have observed this principle for centuries. "The average human brain finds its effective scope in handling from three to six other brains. If a man divides the whole of his work into two branches and delegates his responsibility, freely and properly, to two experienced heads of branches he will not have enough to do. The occasions when they would have to refer to him would be too few to keep him fully occupied. If he delegates to three heads he will be kept fairly busy, whilst six heads of branches will give most bosses a ten-hour day. . . . Of all the ways of waste there is none so vicious as your clever politician trying to run a business concern without having any notion of self-organization. One of them who took over Munitions for a time had so little idea of organizing his own energy that he nearly died of overwork through holding up the work of others; i.e., by delegating responsibility coupled with direct access to himself to seventeen subchiefs. . . . As to whether the groups are three, four, five or six it is useful to bear in mind a by-law; the smaller the responsibility of the group member, the larger may be the number of the group — and vice versa. . . . The nearer we approach the supreme head of the whole organization, the more we ought to work towards groups of three; the closer we get to the foot of the whole organization, the more we work towards groups of six."<sup>1</sup>

It is less well-known in business, though the subject has been mentioned occasionally in management literature. "At a dinner the other evening I heard the President of the General Electric Company asked how many people should report directly to the President of a large industrial company. He said that eight or nine were reporting at present, but that it was too many, and he was reorganizing his functions so that only four or five would report directly to himself; and I imagine that four or five is enough. Not that a chief executive should not have contact with others; but that is about as many general functions as should regularly and directly lead up to him."<sup>2</sup> But as this quotation indicates, the question is regarded as an open one. The principle is not accepted as final, a definite rule which should be followed by all those who seek to administer economically and effectively.

As long as this is so, wastes in organization arising from the multiplication of direct subordinates are likely to continue. Access to the highest possible superior offers opportunities for advancement and is itself in many cases a public acknowledgement of status. Personal ambition, therefore, exercises a constant pressure toward indefinite arrangements and away from clear-cut structure. Superiors who are themselves anxious to enhance their prestige and influence in an organization can do so most readily by increasing their area of control, adding sections and departments to their responsibilities with the least possible emphasis on the organization anomalies thereby created. Strong personalities are sometimes unready to delegate and endeavor to exercise a direct personal supervision over far too many details. Considerations of a political character suggest that

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<sup>1</sup> Sir Ian Hamilton, "The Soul and Body of an Army." Arnold, London, 1921, p.229

<sup>2</sup> H. P. Kendall, "The Problem of the Chief Executive," Bulletin of the Taylor Society, Vol. 7, No. 2, April, 1922

every kind of "interest" should be represented in contact with superior authority. Attempts to simplify situations where a superior has too many subordinates are almost invariably regarded as an attack upon his personal competence.

It is, therefore, of great importance for the art and science of organization that evidence of the validity of the principle as a matter of theory, should be added to the practical experience of those specially interested in organization. In fact, such theoretical evidence is overwhelming. It rests upon two simple considerations. The first is what is known by psychologists as "the span of attention." Generally speaking, in any department of activity the number of separate items to which the human brain can pay attention at the same time is strictly limited. In very exceptional cases, for instance, an individual can memorize groups of figures of more than six digits when read out and can repeat them accurately after a brief interval. But in the vast majority of cases the "span of attention" is limited to six digits. The same holds good of other intellectual activities.

It is, however, the second consideration which has caused the greatest confusion on this question. In almost every case the supervisor measures the burden of his responsibility by the number of direct single relationships between himself and those he supervises. But in addition there are direct group relationships and cross relationships. Thus, if Tom supervises two persons, Dick and Harry, he can speak to each of them individually or he can speak to them as a pair. The behavior of Dick in the presence of Harry or of Harry in the presence of Dick will vary from their behavior when with Tom alone. Further, what Dick thinks of Harry and what Harry thinks of Dick constitute two cross relationships which Tom must keep in mind in arranging any work over which they must collaborate in his absence. The presence of these relationships other than the single direct relationship is not always obvious. Yet the popular expression "he's no good in a crowd" refers to just such changes in human personality as the result of association. And they must constantly, and often simultaneously, constitute additional factors to be controlled.

Thus, even in this extremely simple unit of organization, Tom must hold four to six relationships within his span of attention:

<i>Direct Single Relationships</i>		
Tom to Dick and Tom to Harry.....		2
<i>Direct Group Relationships</i>		
Tom to Dick with Harry and Tom to Harry with Dick.....		2
<i>Cross Relationships</i>		
Harry with Dick and Dick with Harry.....		2
 <i>Total Relationships</i> .....		6

The number of direct and cross relationships will increase in mathematical relation as the number of subordinates assigned to Tom increases. The direct single relationships increase in the same proportion as the number of subordinates assigned. Each person added creates only one single direct relationship. Direct group relationships can be counted either once for each possible combination of subordinates, or once for each individual in each possible combination. Similarly cross relationships can be counted once as "unilateral" or twice as "bilateral." In any event the group and cross relationships increase more rapidly than the number of subordinates assigned, because each fresh individual adds as many more cross and direct group relationships as there are persons

already in the group. Irrespective of the manner of counting, the number of relationships increases in exponential proportion.

The effect of these distinctions as brought out in the accompanying tables and chart, should be read as follows:

- $n$  — number of persons supervised;
- $a$  — number of direct single relationships;
- $b$  — number of cross relationships;
- $c$  — number of direct group relationships;
- $d$  —  $a+b$ ;
- $e$  —  $a+c$ ;
- $f$  —  $a+b+c$

computed on the maximum basis as indicated above. In the second table,  $b'$ ,  $c'$ ,  $d'$ ,  $e'$ , and  $f'$ , indicate similar figures computed on the minimum basis.

Since it is not possible to assign comparable weights to these different varieties of relationship, it is probably safest to accept the most inclusive assumption as the standard by which to judge the relative complexity of supervision imposed by varying numbers of subordinates. This is represented by line  $f$  on the chart. Assuming that it is possible for one supervisor to watch a maximum of 12 cross and 28 direct group relationships, the conclusion follows that, in cases other than routine work, the rapid increase of cross and direct group relationships is the governing factor which actually limits the number of persons which can be effectually and efficiently supervised by one person. Hence the number of lateral divisions in each descending level of responsibility should be restricted to a maximum of five and, most probably, only four.

It is of interest also to note the possible number of different relationships in which the supervisor can stand to any immediate subordinate. On assumption  $f$ , for four subordinates there are eleven relationships with any individual, one direct single, three cross, and seven direct group. In a group of twelve, 2,059 per member. This single fact explains many notorious military disasters. Just why an executive already having four subordinates should hesitate before adding a fifth member to the group which he controls directly, becomes clear if it is realized that the addition not only brings twenty new relationships with him, but adds nine more relationships to each of his colleagues. The total is raised from 44 to 100 possible relationships for the unit, an increase in complexity of 127 per cent in return for a 20 per cent increase in working capacity.

The exception made in a preceding paragraph for cases of routine work, further emphasizes the importance of this principle. It explains at the same time Sir Ian Hamilton's by-law and certain cases where, in apparently successful organizations, larger numbers of subordinates report to a supervisor. It is obvious that, if it is cross and group relationships which introduce complexity into supervision, this factor will operate with much less force where the work done by each of various subordinates does not come into contact with that done by others. This is frequently the case at the lowest level of organization, each worker being given an assigned task involving little or no contact with colleagues. On the other hand, at the higher levels of organization where a large measure of responsibility and freedom is necessarily delegated to subordinates, themselves in charge of important divisions, the number and frequency of cross and group relationships is necessarily much increased.

*Fig. 1. DIRECT AND CROSS RELATIONSHIPS*  
*Table I Computed on Maximum Basis*

relationship		formulae	=	1	2	3	4	5	6	7	8	9	10	11	12
direct single	a =	= n	=	1	2	3	4	5	6	7	8	9	10	11	12
cross	b =	= n (n-1)	=	0	2	6	12	20	30	42	56	72	90	110	132
direct group	c =	= n ( $2^n/2 - 1$ )	=	0	2	9	28	75	186	441	1016	2295	5110	11253	24564
total direct	d = a + b	= $n^2$	=	1	4	9	16	25	36	49	64	81	100	121	144
single and cross	e = a+c	= n ( $2^n/2$ )	=	1	4	12	32	80	192	448	1024	2304	5120	11264	24516
total direct	f = a+b+c	= n (( $2^n/2$ ) + n - 1))	=	1	6	18	44	100	222	490	1080	2376	5210	11374	24708
<i>Table II Computed on Minimum Basis</i>															
direct single	a =	= n	=	1	2	3	4	5	6	7	8	9	10	11	12
cross	b =	= n/2 (n-1)	=	0	1	3	6	10	15	21	28	36	45	55	66
direct group	c =	= $2^n - n - 1$	=	0	1	4	11	26	57	120	247	502	1013	2036	4083
total direct	d = a+b	= $n/2 (n + 1)$	=	1	3	6	10	15	21	28	36	45	55	66	78
single and cross	e = a+c	= $2^n - 1$	=	1	3	7	15	31	63	127	255	511	1023	2047	4095
total direct	f = a+b+c	= $2^n + n/2 (n - 1) - 1$	=	1	4	10	21	41	78	148	283	547	1068	2102	4161

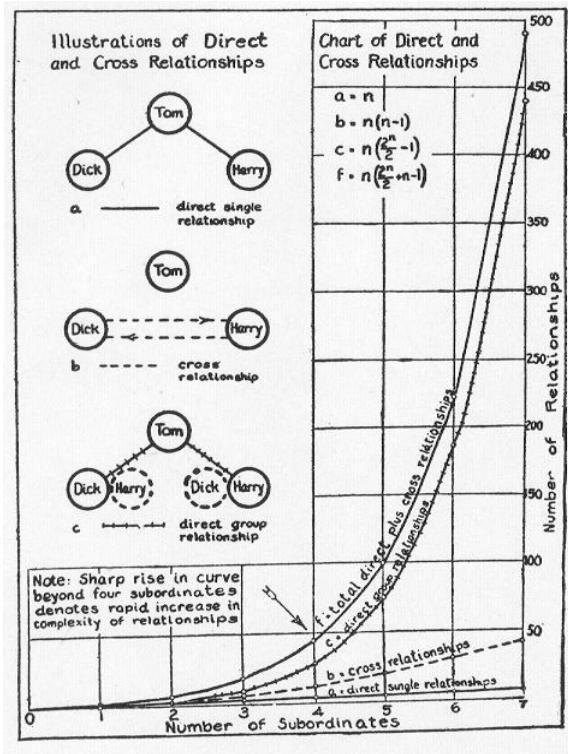
*Fig. 2. CHART SHOWING DIRECT GROUP RELATIONSHIPS*

Number of members per group	1	1 Subordinate A
1	2	2 Subordinates A B AB
1	2	3 Subordinates A B C AB AC BC ABC
1	2	4 Subordinates A B C D AB AC AD BC BD CD ABC ABD ACD BCD ABCD
1	2	5 Subordinates A B C D E AB AC AD AE BC BD BE CD CE DE ABC ABD ABE ABF ACD ACE ACF ADE ADF BCD BCE BDE CDE
3	4	ABC ABCD ABCE ABCF ABDF ABEF ACDE ACDF ACEF ADEF BCDE BCDF BCEF BDEF CDEF
5	6	ABCD E ABCDEF ABCEF ABDEF ACDEF BCDEF ABCDE ABCDF ABCDG ABCFG ABCFG ABDEF ABDEG ABDFG ABEFG ACDEF ACDEG ACDFG ACEFG ADEFG BCDEF BCDFG BCEFG BDEFG CDEFG
7	7	ABCDEF ABCDEG ABCDFG ABCFG ABDEG ACDEG BCDEFG ABCDEG ABCDEG ABCDFG ABCFG ABDEFG ACDEFG BCDEFG ABCDEG

Note that for four subordinates it is quite easy to grasp and remember every combination of groups, but that from five on, this is no longer possible, because the various groups become a maze of confusion.

But even at these higher levels, where immediate subordinates are doing work which does not impinge upon that of their colleagues larger numbers can be controlled.

Fig. 3.



The General in command of a British infantry Division during the Great War had six subordinates reporting directly to him. Three of these were, however, in charge of uniform infantry Brigades who had no regular cross relationships. Similar situations are found in business where a Head Office administers a number of subsidiary companies or branches widely separated geographically and having few relationships of a technical or functional character. But the general evolution of modern business tends towards an increasing degree of specialization. This enforces organization by function with a correspondingly greater demand for co-ordination, creating automatically a wider and wider range of group and cross relationships. The principle which has been discussed is, therefore, likely to become of greater rather than of less importance in all forms of organization.

Note: This is a retyped version of Chapter X in *Papers on the Science of Administration*, edited by Luther Gulick and L. Urwick and published in 1937 by the Institute of Public Administration, Columbia University. The chart immediately above was imaged from the original paper. All else has been rekeyed and is as faithful to the original as modern word processing can make it. When I first retyped this paper, I inadvertently replicated the maximum formulas as the minimum formulas. This was drawn to my attention by Marcin Murawski, a student in Gdansk, Poland and I have since made the necessary corrections.